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# AS Level Further Mathematics A

## Y531/01 Pure Core

**Monday 14 May 2018 – Afternoon**  
**Time allowed: 1 hour 15 minutes**



**You must have:**

- Printed Answer Booklet
- Formulae AS Level Further Mathematics A

**You may use:**

- a scientific or graphical calculator

### INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . Unless otherwise instructed, when a numerical value is needed, use  $g = 9.8$ .

### INFORMATION

- The total mark for this paper is **60**.
- The marks for each question are shown in brackets [ ].
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **4** pages.

Answer **all** the questions.

- 1 (i) Find a vector which is perpendicular to both  $\begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$  and  $\begin{pmatrix} -3 \\ -6 \\ 4 \end{pmatrix}$ . [2]

1i) Using the cross product

$$\begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} \times \begin{pmatrix} -3 \\ -6 \\ 4 \end{pmatrix} = \begin{pmatrix} i & j & k \\ 1 & 3 & -2 \\ -3 & -6 & 4 \end{pmatrix} = \begin{pmatrix} 12 - 12 \\ -(4 - 6) \\ -6 + 9 \end{pmatrix} \\ = \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}$$

OR

Using simultaneous equations

$$\begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$$

$$x + 3y - 2z = 0$$

$$\begin{pmatrix} -3 \\ -6 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$$

$$-3x - 6y + 4z = 0$$

$$\begin{aligned} \text{let } y &= 2 \\ x + 3(2) - 2z &= 0 \\ x - 2z &= -6 \quad \textcircled{1} \end{aligned}$$

$$\begin{aligned} -3x - 6(2) + 4z &= 0 \\ -3x + 4z &= 12 \quad \textcircled{2} \end{aligned}$$

$$2 \times \textcircled{1} + \textcircled{2}$$

$$\begin{aligned} 2x - 4z &= -12 \\ + \underline{-3x + 4z} &= 12 \end{aligned}$$

$$-x = 0$$

$$x = 0$$

$\therefore$  when  $x=0$ ,  $y=2$ , sub into  $x+3y-2z=0$

$$0 + 3(2) - 2z = 0$$

$$6 = 2z$$

$$3 = z$$

$$\therefore \text{ vector} = \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}$$

(ii) The cartesian equation of a line is  $\frac{x}{2} = y - 3 = 2z + 4$ .

Express the equation of this line in vector form.

[3]

$$\frac{x}{2} = y - 3 = 2z + 4$$

$$\frac{x-0}{2} = \frac{y-3}{1} = \frac{z+2}{\frac{1}{2}}$$

$$\underline{r} = \begin{pmatrix} 0 \\ 3 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ \frac{1}{2} \end{pmatrix}$$

2 In this question you must show detailed reasoning.

The cubic equation  $2x^3 + 3x^2 - 5x + 4 = 0$  has roots  $\alpha$ ,  $\beta$  and  $\gamma$ . By making an appropriate substitution, or otherwise, find a cubic equation with integer coefficients whose roots are  $\frac{1}{\alpha}$ ,  $\frac{1}{\beta}$  and  $\frac{1}{\gamma}$ . [3]

$$\text{let } u = \frac{1}{x} \Rightarrow \frac{1}{u} = x$$

$$2\left(\frac{1}{u}\right)^3 + 3\left(\frac{1}{u}\right)^2 - 5\left(\frac{1}{u}\right) + 4 = 0$$

$$\frac{2}{u^3} + \frac{3}{u^2} - \frac{5}{u} + 4 = 0 \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \times u^3$$

$$2 + 3u - 5u^2 + 4u^3 = 0$$

$$4u^3 - 5u^2 + 3u + 2 = 0$$

OR

$$\alpha + \beta + \gamma = -\frac{b}{a} = -\frac{3}{2}$$

$$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a} = -\frac{5}{2}$$

$$\alpha\beta\gamma = \frac{-d}{a} = -\frac{4}{2} = -2$$

$\therefore$  as roots are  $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$

$$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\alpha\beta + \beta\gamma + \alpha\gamma}{\alpha\beta\gamma} = \frac{-\frac{5}{2}}{-2} = \frac{5}{4}$$

$$\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\alpha\gamma} = \frac{\gamma + \alpha + \beta}{\alpha\beta\gamma} = \frac{-\frac{3}{2}}{-2} = \frac{3}{4}$$

$$\frac{1}{\alpha\beta\gamma} = -\frac{1}{2}$$

$$\therefore u^3 - \frac{5}{4}u^2 + \frac{3}{4}u + \frac{1}{2} = 0$$

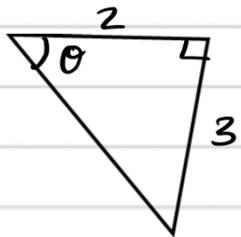
$$\Rightarrow 4u^3 - 5u^2 + 3u + 2 = 0$$

3 In this question you must show detailed reasoning.

The complex numbers  $z_1$  and  $z_2$  are given by  $z_1 = 2 - 3i$  and  $z_2 = a + 4i$  where  $a$  is a real number.

- (i) Express  $z_1$  in modulus-argument form, giving the modulus in exact form and the argument correct to 3 significant figures. [3]

$$|z_1| = \sqrt{2^2 + (-3)^2}$$
$$= \sqrt{13}$$



$$\arg z_1 = \theta$$
$$\arctan\left(-\frac{3}{2}\right) = -0.983 \quad (3\text{sf})$$

$$z_1 = \sqrt{13} (\cos(-0.983) + i \sin(-0.983))$$

$$z_1 = \sqrt{13} (\cos(0.983) - i \sin(0.983))$$

- (ii) Find  $z_1 z_2$  in terms of  $a$ , writing your answer in the form  $c + id$ . [2]

$$(2 - 3i)(a + 4i) = 2a - 3ai + 8i - 12i^2$$
$$= (2a + 12) + i(8 - 3a)$$

- (iii) The real and imaginary parts of a complex number on an Argand diagram are  $x$  and  $y$  respectively. Given that the point representing  $z_1 z_2$  lies on the line  $y = x$ , find the value of  $a$ . [2]

$$y = x$$
$$8 - 3a = 2a + 12$$
$$-4 = 5a$$
$$a = -\frac{4}{5}$$

- (iv) Given instead that  $z_1 z_2 = (z_1 z_2)^*$  find the value of  $a$ . [2]

$$\text{Imaginary part} = 0$$
$$8 - 3a = 0$$
$$8 = 3a$$
$$a = \frac{8}{3}$$

4 The matrix A is given by  $A = \begin{pmatrix} 2 & 1 & 2 \\ 1 & -1 & 1 \\ 2 & 2 & a \end{pmatrix}$ .

(i) Show that  $\det A = 6 - 3a$ .

[2]

$$\begin{aligned} \det A &= 2(-a-2) - (a-2) + 2(2+2) \\ &= -2a-4 - a+2 + 8 \\ &= -3a+6 \\ &= 6-3a \end{aligned}$$

(ii) State the value of  $a$  for which A is singular.

[1]

$$\begin{aligned} \det A &= 0 \\ 6 - 3a &= 0 \\ 6 &= 3a \\ \underline{2} &= \underline{a} \end{aligned}$$

(iii) Given that A is non-singular find  $A^{-1}$  in terms of  $a$ .

[4]

$$\text{Matrix of cofactors} = \begin{pmatrix} -a-2 & 2-a & 4 \\ 4-a & 2a-4 & -2 \\ 3 & 0 & -3 \end{pmatrix}$$

$$\text{Transpose} = \begin{pmatrix} -a-2 & 4-a & 3 \\ 2-a & 2a-4 & 0 \\ 4 & -2 & -3 \end{pmatrix}$$

$$A^{-1} = \frac{1}{6-3a} \begin{pmatrix} -a-2 & 4-a & 3 \\ 2-a & 2a-4 & 0 \\ 4 & -2 & -3 \end{pmatrix}$$

5 In this question you must show detailed reasoning.

(i) Express  $(2+3i)^3$  in the form  $a+ib$ .

[3]

$$\begin{aligned}(2+3i)^3 &= 2^3 + 3(2)^2(3i) + 3(2)(3i)^2 + (3i)^3 \\ &= 8 + 36i + 54i^2 + 27i^3 \\ &= 8 + 36i - 54 - 27i \\ &= -46 + 9i\end{aligned}$$

(ii) Hence verify that  $2+3i$  is a root of the equation  $3z^3 - 8z^2 + 23z + 52 = 0$ .

[3]

$$\begin{aligned}3(2+3i)^3 - 8(2+3i)^2 + 23(2+3i) + 52 \\ = 3(-46+9i) - 8(4+12i-9) + 46 + 69i + 52 \\ = -138 + 27i - 8(12i-5) + 98 + 69i \\ = -40 + 96i - 96i + 40 \\ = 0\end{aligned}$$

Hence  $2+3i$  is a root.

(iii) Express  $3z^3 - 8z^2 + 23z + 52$  as the product of a linear factor and a quadratic factor with real coefficients. [4]

If  $2+3i$  is a root, then  $2-3i$  is also a root as it is its conjugate pair.

This means  $(z-(2+3i))$  and  $(z-(2-3i))$  are factors.

$$\begin{aligned}(z-(2+3i))(z-(2-3i)) \\ = z^2 - 2z + 3zi - 2z + 4 - 6i - 3zi + 6i - 9i^2 \\ = z^2 - 4z + 4 + 9 \\ = z^2 - 4z + 13\end{aligned}$$

$$\begin{aligned}(z^2 - 4z + 13)(az + b) \\ = az^3 + bz^2 - 4az^2 - 4bz + 13az + 13b\end{aligned}$$

compare coefficients

$$z^3 \text{ coeff. : } a = 3$$

$$z^2 \text{ coeff. : } b - 4a = -8$$

$$b - 4(3) = -8$$

$$b - 12 = -8$$

$$b = 4$$

$$\therefore (z^2 - 4z + 13)(3z + 4)$$

OR

$$\begin{array}{r} z^2 - 4z + 13 \overline{) 3z^3 - 8z^2 + 23z + 52} \\ \underline{- 3z^3 - 12z^2 + 39z} \phantom{+ 52} \\ \phantom{z^2 - 4z + 13} + 4z^2 - 16z + 52 \\ \underline{- 4z^2 - 16z + 52} \\ \phantom{z^2 - 4z + 13} \phantom{+ 4z^2 - 16z} 0 \end{array}$$

$$\therefore (z^2 - 4z + 13)(3z + 4)$$

6 The matrices **A** and **B** are given by  $A = \begin{pmatrix} t & 6 \\ t & -2 \end{pmatrix}$  and  $B = \begin{pmatrix} 2t & 4 \\ t & -2 \end{pmatrix}$  where  $t$  is a constant.

(i) Show that  $|A| = |B|$ .

[2]

$$|A| = -2t - 6t = -8t$$

$$|B| = -4t - 4t = -8t$$

$$\therefore |A| = |B|$$

(ii) Verify that  $|AB| = |A||B|$ .

[3]

$$AB = \begin{pmatrix} t & 6 \\ t & -2 \end{pmatrix} \begin{pmatrix} 2t & 4 \\ t & -2 \end{pmatrix}$$

$$|A||B| = 8t \times 8t = 64t^2$$

$$= \begin{pmatrix} 2t^2 + 6t & 4t - 12 \\ 2t^2 - 2t & 4t + 4 \end{pmatrix}$$

$$\text{As } 64t^2 = 64t^2 \\ \text{then } |A||B| = |AB|$$

$$\begin{aligned} |AB| &= (2t^2 + 6t)(4t + 4) - (2t^2 - 2t)(4t - 12) \\ &= 8t^3 + 24t^2 + 8t + 24t - 8t^3 + 24t^2 + 8t^2 - 24t \\ &= 64t^2 \end{aligned}$$

(iii) Given that  $|AB| = -1$  explain what this means about the constant  $t$ .

[2]

$$64t^2 = -1$$

$$t^2 = -\frac{1}{64}$$

$$t = \pm \frac{i}{8}$$

$\therefore t$  is imaginary.

when  $n=1$

$$\begin{aligned} 2^{1+1} + 5 \times 9^1 &= 2^2 + 5 \times 9 \\ &= 4 + 45 \\ &= 49 \\ &= 7 \times 7 \end{aligned}$$

As 49 is divisible by 7, then when  $n=1$ , it is divisible by 7.

Assume  $n=k$  is divisible by 7.

$f(k) = 2^{k+1} + 5 \times 9^k$  is divisible by 7.

When  $n=k+1$

$$2^{k+1+1} + 5 \times 9^{k+1}$$

$$\begin{aligned} f(k+1) &= 2 \times 2^{k+1} + 5 \times 9 \times 9^k \\ &= 2 \times 2^{k+1} + 7 \times 5 \times 9^k + 2 \times 5 \times 9^k \\ &= 2(2^{k+1} + 5 \times 9^k) + 7 \times 5 \times 9^k \\ &= 2f(k) + 7 \times 5 \times 9^k \end{aligned}$$

So true for  $n=k \therefore$  true for  $n=k+1$ . But true for  $n=1$ . So true for all positive integers  $n \geq 1$ .

8 The  $2 \times 2$  matrix  $A$  represents a transformation  $T$  which has the following properties.

- The image of the point  $(0, 1)$  is the point  $(3, 4)$ .
- An object shape whose area is 7 is transformed to an image shape whose area is 35.
- $T$  has a line of invariant points.

(i) Find a possible matrix for  $A$ .

[8]

$$\text{let } A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

$$b = 3, d = 4.$$

The area has been scaled by 5, meaning  $\det A = 5$

$$ad - bc = 5$$

$$4a - 3c = 5 \quad \text{--- (1)}$$

Invariant points:  $\begin{pmatrix} a & 3 \\ c & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$

$$ax + 3y = x$$

$$x(1-a) = 3y$$

$$cx + 4y = y$$

$$3y = -cx$$

Set these two equations equal to each other:

$$x(1-a) = -cx$$

$$1-a = -c$$

$$1+c = a$$

Sub this into equation ①

$$4(1+c) - 3c = 5$$

$$4 + 4c - 3c = 5$$

$$4 + c = 5$$

$$c = 1$$

$$\therefore a = 1 + 1 = 2$$

$$\therefore A = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$$

The transformation S is represented by the matrix B where  $B = \begin{pmatrix} 3 & 1 \\ 2 & 2 \end{pmatrix}$ .

(ii) Find the equation of the line of invariant points of S. [2]

$$\begin{pmatrix} 3 & 1 \\ 2 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$3x + y = x$$

$$y = -2x$$

$$2x + 2y = y$$

$$y = -2x$$

$\therefore$  invariant line is  $y = -2x$

(iii) Show that any line of the form  $y = x + c$  is an invariant line of S. [3]

$$\begin{pmatrix} 3 & 1 \\ 2 & 2 \end{pmatrix} \begin{pmatrix} x \\ x+c \end{pmatrix} = \begin{pmatrix} 3x + x + c \\ 2x + 2x + 2c \end{pmatrix}$$

$$= \begin{pmatrix} 4x + c \\ 4x + 2c \end{pmatrix}$$

Hence the line  $y = x + c$  is invariant under S for all c.

This is also in the form  $y = x + c$

$$4x + 2c = (4x + c) + c$$

$$4x + 2c = 4x + 2c.$$

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